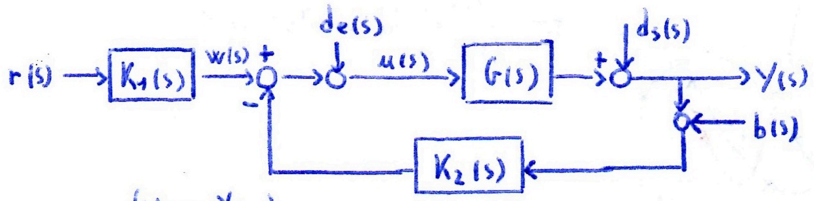
Classical Control

Ecole Centrale Nantes - 2018/19 - by Davide Lanza

[1.]



$L(s) = G(s)\,K_2(s)$

$S(s) = \dfrac{1}{1+L(s)}$

$T(s) = 1 - S(s) = \dfrac{L(s)}{1+L(s)}$ TRANSFERT DE BOUCLE

SISO SYS.
$$\begin{cases} y(s) = \underbrace{[G K_1 S(s)]}_{(H(s) = Y/r)} r(s) + [G S(s)]\, d_e(s) + [S(s)]\, d_s(s) + [T(s)]\, b(s) \\ u(s) = [K_1 S(s)]\, r(s) + [S(s)]\, d_e(s) + [K_2 S(s)]\, d_s(s) + [K_2 S(s)]\, b(s) \end{cases}$$

$$\frac{\Delta H(s)}{H(s)} = \underbrace{S_{nom}(s)}_{(S(s) \ll 1 \text{ GOOD})} \frac{\Delta G(s)}{G(s)}$$

(K_2 -> STABILITY (NOM & ROBUST)
K_2 -> PERF. IN REGULATION
K_1 -> PERF. IN FOLLOWING)

$$\left[\begin{aligned} 20 \log_{10}(X) &= X_{dB} \\ 10^{X_{dB}/20} &= X \end{aligned}\right]$$

$$\left[\begin{aligned} |z| &= \sqrt{x^2+y^2} \\ \arg(z) &= \begin{cases} \operatorname{atan}(Y/X), & X>0 \\ \operatorname{atan}(Y/X)+\pi, & X<0 \quad Y \geq 0 \\ \operatorname{atan}(Y/X)-\pi, & X<0 \quad Y<0 \\ \pm\pi/2, & X=0 \quad Y \gtrless 0 \\ \text{INDET}, & X=0 \quad Y=0 \end{cases} \end{aligned}\right]$$

[2. STABILITY]

COND. NEC. : STABILITY $\Leftrightarrow$ in $\left(P(s) = \sum_n a_n s^n\right)$ $a_n > 0 \;\forall n$

Routh:

s^n	a_0	a_2	a_4 ...
s^{n-1}	a_1	a_3	a_5 ...
$\vdots$	b_1	b_2	

[ANTI-DETERMINANT
$\begin{matrix} a_0 & a_2 \\ a_1 & a_3 \end{matrix}$ $\qquad$ $\begin{matrix} a_0 & a_4 \\ a_1 & a_5 \end{matrix}$

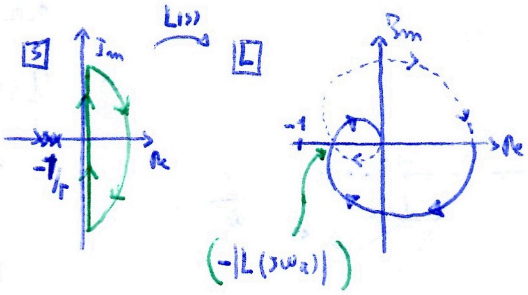
$\dfrac{a_1 a_2 - a_0 a_3}{a_1} = b_1$ $\qquad$ b_2]

IF ALL ROW $\emptyset$ -> THERE IS A SIMMETRY
$P_K(s) = K_1 s^n + \dots + K_{n-1} s + K_n = \emptyset$
COEFF. "K_i" OF THE PREVIOUS ROW
↓
DERIVE $P_K(s)$ -> (COEFF OF DERIVATE IN THE ACTUAL ROW)

↓
(ALL POLE STABLES ($Re < \emptyset$) $\Leftrightarrow$ NO SIGN CHANGES IN FIRST COLUMN)

Nyquist:

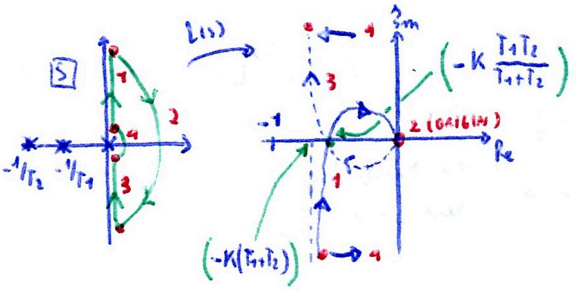
- $L(s) = \dfrac{K}{(1+sT)^3}$



-> STABLE IF $|L(j\omega_\pi)| < 1$ $(N_A = \emptyset)$
-> UNSTABLE IF $|L(j\omega_\pi)| > 1$ $(N_A = 2)$

$Z^+_{OL} = P^+_{OL} - N_A$
(P^+_{CL}) (ANTI-CLOCKWISE)

- $L(s) = \dfrac{K}{s(1+sT_1)(1+sT_2)}$



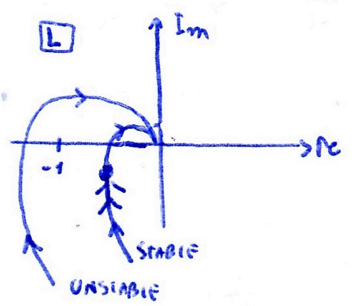
-> STABLE IF $-K \dfrac{T_1 T_2}{T_1+T_2} > -1$ $(N_A = \emptyset)$

-> UNSTABLE IF $-K \dfrac{T_1 T_2}{T_1+T_2} < -1$ $(N_A = 2)$

Simplified Nyquist:

Hyp 1: $\nexists P^+_{OL} \wedge \nexists$ Poles $Re = \emptyset$ with multiplicity > 1
Hyp 2: sys. Minimum Phase
↳ STABLE IF (-1) AT LEFT !

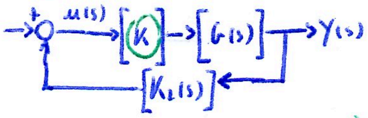
->

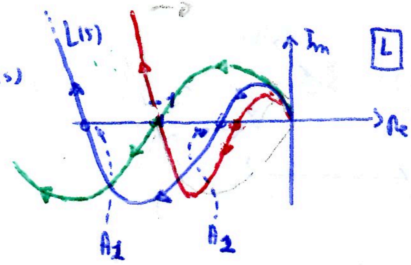


-> (IN NICHOLS PLOT (-1) AT RIGHT !)

2.4 Robust Stability

Gain Margin:





L

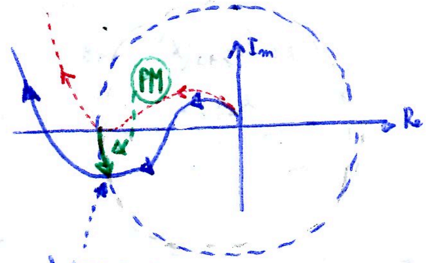
($|A_1|, |A_2|$ are $|L(\omega_{\pi i})|$, $i=1,2$)

(In the figure L(s) ~~[illegible]~~ ensures stability)

$\frac{L(s)}{A_1} \to \frac{L(s)}{A_1}$ on (-1) $\frac{L(s)}{A_2} \to \frac{L(s)}{A_2}$ on (-1) $\Rightarrow$ $[M_{g1}, M_{g2}] = \left[20 \cdot \log_{10}\left(\frac{1}{|A_1|}\right), 20 \cdot \log_{10}\left(\frac{1}{|A_2|}\right)\right]$

Normally: $[-6\,dB, +6\,dB]$

Phase Margin: $(\dots) \to [e^{-j\varphi}] \to (\dots)$



$M_\varphi \equiv PM = 180 + \angle L(j\omega_c)$

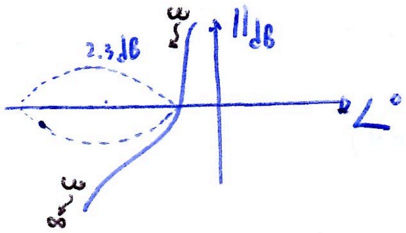
M_φ OK IF

ω_φ is ω_c

IN RADIANTS!

Delay Margin: $(\dots) \to [e^{-sT}] \to (\dots)$ $DM \equiv M_d = \frac{M_\varphi}{\omega_\varphi}$

M-Circles:



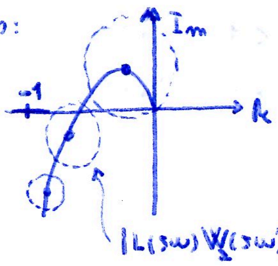
$M_r = \frac{|T(j\omega)|_{max}}{|T(0)|} = 2.3\,dB$

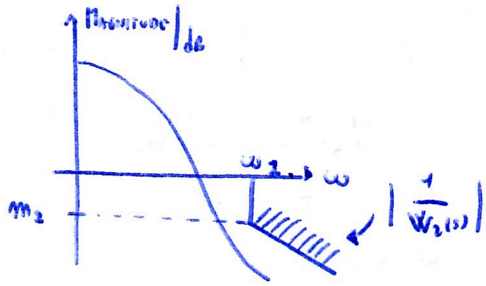
(Shortest distance from -1: $|L(j\omega) - (-1)|$)

$M_{sd} = \frac{1}{\|S(s)\|_{max}}$ (shortest distance)

If $M_{sd} > 1$:
- $\to \|S(s)\|_\infty < 1$
- $\to M_{g1} \le -6\,dB$, $M_{g2} = \infty$
- $\to M_\varphi > 60°$
- $\to M_d \geq \frac{\pi}{3}\frac{1}{\omega}$

Templates:





Robust stability template:

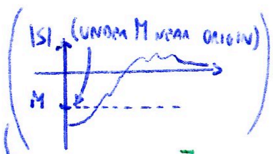
$\Rightarrow |L(j\omega)| \le \frac{1}{|W_2(j\omega)|}$

$\forall \omega \in [\omega_2, +\infty]$

3. Performances

Precision → static/asymptotic performance: regulation $[d(t)\ \&\ b(t)]$, tracking $[r(t)]$ } index: ($e_\infty(t)$ static error, static gain of L(s)) → $\tilde{L}(\emptyset)$ → ($\tilde{L}$ is $L(s)$ without poles in $\emptyset$)

Rapidity → dynamic performances: regulation, tracking } index: ($t_p, t_{r(5\%)}, t_{r(2\%)}, \dots$, ω_c, ω_r)



- Static regulation performances:

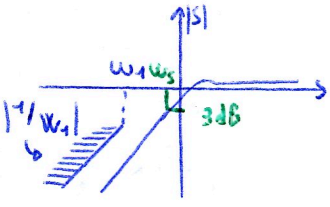
$\left(\frac{u}{d_e}(s) = \frac{y}{d_s}(s) = S(s) = \frac{1}{1+L(s)}\right) \Rightarrow \lim_{s\to\emptyset} |s\,S(s)\,d(s)| < M \to \left[|S(s)|_{s=j\omega} \le \left|\frac{M}{s\,d(s)}\right|_{s=j\omega}\right] \leftarrow$ (near the origin)

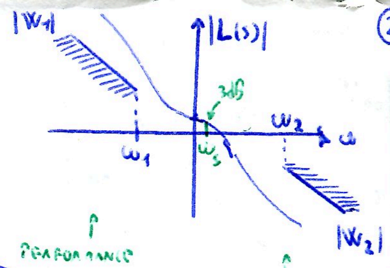
Class of d(s) \ L(s)	∅	1	2	3
$d(s) = \frac{d}{s}$	$e_\infty = \left\lvert\frac{1}{1+L(0)}\right\rvert$	∅	∅	∅
$d(s) = \frac{d}{s^2}$	∞	$e_\infty = \left\lvert\frac{1}{\tilde{L}(0)}\right\rvert$	∅	∅
$d(s) = \frac{d}{s^3}$	∞	∞	$e_\infty = \left\lvert\frac{1}{\tilde{L}(0)}\right\rvert$	∅

Perfect static regulation $(M = \emptyset) \iff class(L(s)) \geq class(d(s))$

- DYNAMIC REGULATION PERFORMANCES

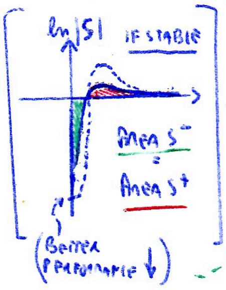
$$\left[|S(j\omega)|_{(s=j\omega)} \leq \left|\frac{1}{W_1(s)}\right|_{(s=j\omega)} \quad \text{for } \omega \in [\emptyset, \omega_1) \right] \to \left(L(s) \geq W_1(s) \right)$$





↳ QUANTITATIVE INDEX : [BAND WIDTH $\omega_s \triangleq \omega$ s.t. $|S(j\omega)| = -3\,dB$ for the first time]

↳ TH. BODE FREUDENBERG: $\left[\begin{array}{l} L(s) \text{ STABLE} \Rightarrow \int_\emptyset^\infty \ln|S(j\omega)|\,d\omega = \emptyset \\ L(s) \text{ INSTABLE} \Rightarrow \int_\emptyset^\infty \ln|S(j\omega)|\,d\omega = \pi \sum_i Re(p_i) \end{array} \right] \Rightarrow$

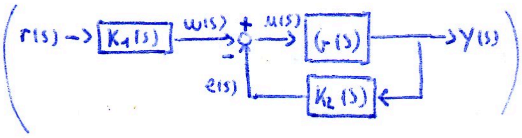


$\to M_{sd} = \inf_\omega |L(j\omega)+1| = \frac{1}{|S_{max}|}$

↳ FOR PERFORMANCE → HIGH PEAK

↳ FOR STABILITY → LOW PEAK

- STATIC TRACKING PERFORMANCES:



$\to \left(R(s) \to [K_1] \to [G \cdot S(s)] \to Y(s) \right)$

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} r(t) \quad \Rightarrow \left[K_1(s) = \frac{1}{G(\emptyset)S(\emptyset)} = \frac{1 + G(\emptyset)K_2(\emptyset)}{G(\emptyset)} \right] \text{ (CONSTANT VALUE)}$$

- IF $G(s) = \frac{\tilde{G}(s)}{s^3} \Rightarrow K_1(s) = K_2(\emptyset)$
- IF $S(\emptyset) = \emptyset \Rightarrow K_1(s) = \frac{\tilde{K}_2(\emptyset)}{s^2} \to$ [TO SIMPLIFY $s=\emptyset$ POLES OF $S(s)$]

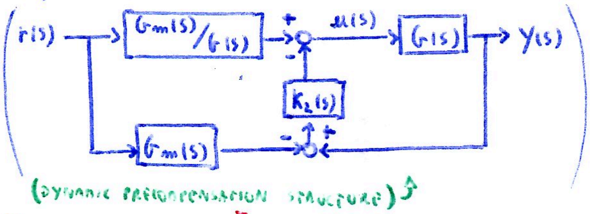
- DYNAMIC TRACKING PERFORMANCES:

$\left(\cancel{K_1(s) = \frac{1}{G(s)S(s)} = \frac{1}{G(s)} + K_2(s)} \text{ NO} \right) \Rightarrow K_1(s) = \frac{G_m(s)}{G(s)S(s)} = \frac{G_m(s)}{G(s)} + K_2(s)\,G_m(s)$

2 CRITERIA → [1) (CANCEL $Re \geq \emptyset$ ZEROS) 2) (G_m deg $\geq$ G deg)

ex: $\frac{(s-1)(s+10)}{s^2(s+100)} = G(s) \to \deg = 1$

$\frac{(s-1) \cdot Num}{Denom.} = G_m(s) \to \deg = 2$]



(DYNAMIC PRECOMPENSATION STRUCTURE)

(BUT : $Y/_R = G_m(s)$)

$$G_m(s) = \frac{B^+(s)}{(1 + sT_r)^{n_r}}$$

[CONTAINS THE ZEROS WITH $Re \geq \emptyset$ AND $B^+(\emptyset) = 1$]

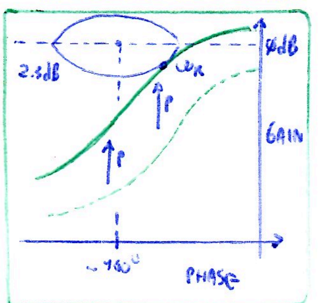
[n_r s.t. $G_m/G(s)$ AT LEAST JUST PROPER ($\deg = \emptyset$)]

[T_r is the only DoF that we can set:
(T_r ↑) ⇒ ACCEPTABLE CONTROL (AVOID SATURATION)
(T_r ↓) ⇒ BETTER TIME RESPONSE]

A. PID CONTROLLERS

$$K(s) = P\left(1 + \frac{1}{sT_i} + \frac{sT_d}{1+sT}\right) \text{ (IN REAL LIFE)}$$

- PROPORTIONAL CONTROLLER : $K(s) = P$ → [LIMITED GAIN INCREASE]



↓ STABILITY MARGINS ↓ STATIC ERROR (HIGHER GAIN)

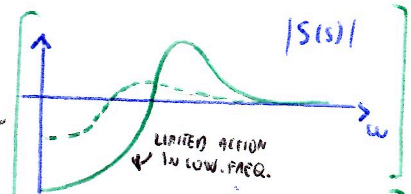
↓ STATIC DISTURBANCE ERROR (↓ |S(∅)|)

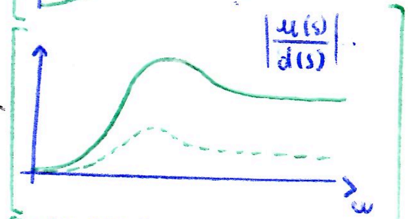
↓ REACTION TIMING ↑ OSCILLATIONS (↑ BANDWIDTH OF $S(j\omega)$)

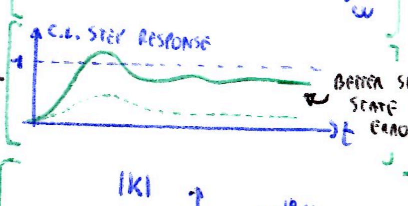
↑ COMMAND SIGNAL AMPLITUDE

↓ |S(s)| IN LOW. FREQUENCIES

↑ SENSIBILITY OF $u(t)$ TO DISTURBS $d(t)$

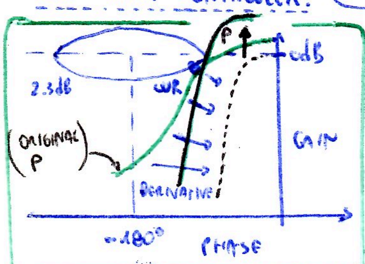






BETTER STEADY STATE ERROR

- P. DERIVATIVE CONTROLLER: $K_s = P(1 + sT_d)$ → [FAST MODE, STABILIZING EFFECT (PHASE LEAD) LARGE $u(t)$ → HIGH FREQ. ERRORS]

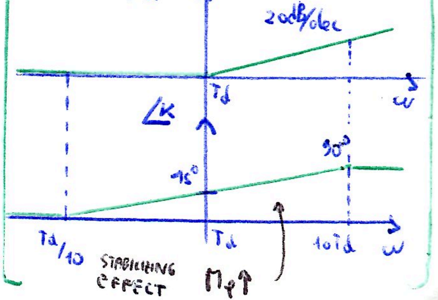


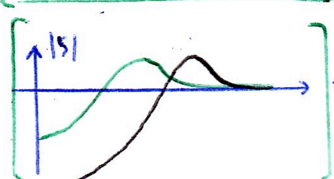
$$\frac{1}{\omega_R} < T_d < \frac{10}{\omega_R} \quad \text{(CRITERION)}$$

LARGE CONTROL SIGNALS (HIGHER GAINS IN HIGH FREQUENCY)

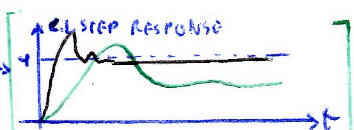
↳ THE FILTER IS USEFUL → $K(s) = P\left(1 + \frac{sT_d}{1+sT}\right)$

↳ NORMALLY $\left[T = T_d/_{10}\right]$

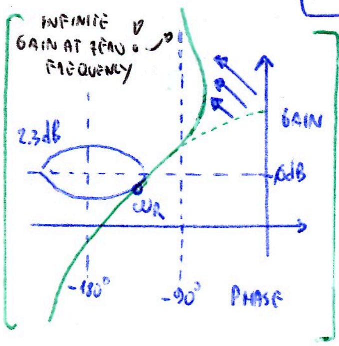




[AS BEFORE: HIGHER GAIN ↑ BANDWIDTH (SEE PROPORTIONAL)]

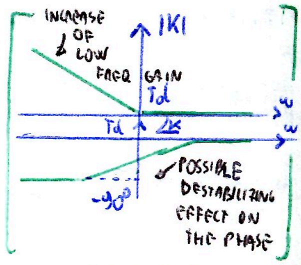


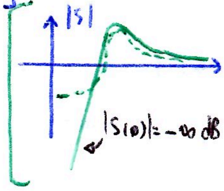
- **P. INTEGRAL CONTROLLER**: $\boxed{K(s) = P\left(1 + \frac{1}{sT_i}\right)}$ → [SLOW REACTION, PERFECT INVERSION AT $\omega = \emptyset$ ($\varepsilon_\infty = \emptyset$)
POLE IN $\emptyset$ → DETRIMENTAL FOR LOOP STABILITY (LOW FREQ.)
IF THERE IS ACTUATOR LIMITATION IT COULD LEAD TO SATURATION]

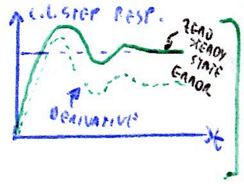


(CRITERION) → $\frac{1}{\omega_R} < T_i < \frac{10}{\omega_R}$ → [WITH THIS NICHOLS CROSS A LITTLE THE M-CIRCLE
↳ YOU HAVE TO DECREASE A LITTLE P]

→ $T_i = \frac{10}{\omega_R}$ BETTER







Summary → P.I.D. Controller

[
P: GLOBAL HIGH GAINS

D: STABILIZING EFFECT OR PHASE LEAD → HIGHER GAIN POSSIBILITY (CONS: YIELDS LARGE $u(t)$ TO HIGH FREQ ERRORS)

I: INFINITE GAIN IN LOW FREQ → ZERO STEADY STEP ERROR (CONS: POLE IN $\emptyset$ → DESTABILIZING EFFECT AND COULD LEAD TO SATURATION)
]

- **ZIEGLER - NICHOLS TUNING METHOD**:

OSCILLATION METHOD:

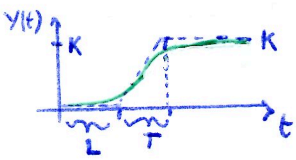
$$\left(\xrightarrow{+} \bigcirc \rightarrow [P_c \uparrow] \rightarrow [G(s)] \rightarrow y \right)$$

↳ $\left[G(s) = K \frac{e^{-Ls}}{1+sT} \right]$

INCREASE P_c UNTIL $u(t)$ START OSCILLATING
↳ RECORD THE CRITICAL P_c AND THE OSCILLATION PERIOD T_c
↓
(IS VERY SENSITIVE TO $\left(\frac{L}{T}\right)$ RATIO)

P	T_i	T_d	
$(0.5)P_c$	X	X	P
$(0.45)P_c$	$(0.83)T_c$	X	PI
$(0.6)P_c$	$(0.5)T_c$	$(0.125)T_c$	PID

REACTION CURVE BASED MODELS:



$G(s) = K \frac{e^{-Ls}}{1+sT}$ $\quad K(s) = P\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1+sT}\right)$

→ (DAMPING $\xi = 0.21$)

P	T_i	T_d	
$\frac{T}{KL}$	X	X	P
$\frac{0.9T}{KL}$	3.3L	X	PI
$\frac{1.2T}{KL}$	2L	$\frac{L}{2}$	PID